<u>Exercise 2</u>
Mathematical basics

Task 1: Binary number system (Points: 12)

a) Briefly describe how numbers can be converted from the decimal system to the binary system. Also briefly describe the reverse direction: converting a number from the binary system to the decimal system.

<u>Answer</u>
Converting decimal to binary

1. The number in base 10 is divided by 2.
2. Assign the quotient for the next iteration and divide by 2.
3. Assign the remainder from as the binary digit.
4. Iterate the steps until the quotient result is finally less than 2.
5. Read the remainders from the last quotient through the remainders bottom to top to get results in base 2.

Converting binary to decimal

1. Write down the binary number and assign the number position (0 - 7) to each digit.
2. Write each digit as (digit x 2)$^{\text{position of number}}$
3. Starting with the least significant digit (from the right), multiply the digit by the value of the position. Continue doing this until the most significant is reached.
4. Perform an addition of all the results and get the decimal equivalent of the given binary number.

b) Convert the following numbers from the decimal system to the binary system. Indicate your calculation method for all subtasks.

[1] 3_{10} = $\mathbf{11_2}$

Division by 2	**Quotient**	**Reminder**	**Bit**
3/2	1	1	0
1/2	0	1	1

[2] 73_{10} = **1001001_2**

Division by 2	Quotient	Reminder	Bit
73/2	36	1	0
36/2	18	0	1
18/2	9	0	2
9/2	4	1	3
4/2	2	0	4
2/2	1	0	5
1/2	0	1	6

[3] 4096_{10} = **1000000000000_2**

Division by 2	**Quotient**	**Reminder**	**Bit**
4096/2	2048	0	0
2048/2	1024	0	1
1024/2	512	0	2
512/2	256	0	3
256/2	128	0	4
128/2	64	0	5
64/2	32	0	6
32/2	16	0	7
16/2	8	0	8
8/2	4	0	9
4/2	2	0	10
2/2	1	0	11
½	0	1	12

[4] $26,2_{10}$ (Cancel in the decimal point area if the sequence repeats.)

26_{10} + 0.2_{10} = 11010.0011_2

- (26_{10}) Read Bits in Descending order

Division by 2	**Quotient**	**Reminder**	**Bit**
26/2	13	0	0
13/2	6	1	1
6/2	3	0	2
3/2	1	1	3
1/2	0	1	4

- (0.2_{10}) Read Bits in Ascending order

Multiplication by 2	Remainder	Bit
0.2 x 2 = 0.4	0.4	0
0.4 x 2 = 0.8	0.8	0
0.8 x 2 = 1.6	0.6	1
0.6 x 2 = 1.2	0.2	1
0.2 x 2 = 0.4	**0.4**	**0 (It starts repeating from here)**

c) Convert the following numbers from the binary system to the decimal system. Indicate your calculation method for all subtasks.

[1] $11101_2 = 29_{10}$
$(11101)_2 = (1 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = \mathbf{(29)_{10}}$

[2] $00010_2 = 2_{10}$
$(00010)_2 = (0 \times 2^4) + (0 \times 2^3) + (0 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = \mathbf{(2)_{10}}$

[3] $1001110_2 = 78_{10}$
$(1001110)_2 = (1 \times 2^6) + (0 \times 2^5) + (0 \times 2^4) + (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = \mathbf{(78)_{10}}$

[4] $1001{,}110_2 = 9.75_{10}$
$(1001.110)_2 = (1 \times 2^3) + (0 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) + (1 \times 2^{-1}) + (1 \times 2^{-2}) + (0 \times 2^{-3}) = \mathbf{(9.75)_{10}}$

d) Perform addition, subtraction, multiplication, and division in the binary system:
Addition
[1] 0 + 0 = 0
[2] 0 + 1 = 1
[3] 1 + 0 = 1
[4] 1 + 1 = 10

Subtraction
[5] 0 - 0 = 0
[6] 0 - 1 = 1
[7] 1 - 0 = 1
[8] 1 - 1 = 0

Multiplication
[9] 0 * 0 = 0
[10] 0 * 1 = 0
[11] 1 * 0 = 0
[12] 1 * 1 = 1

Division
[13] 0 : 0 = 0
[14] 0 : 1 = 0
[15] 1 : 0 = **Undefined**
[16] 1 : 1 = 1

e) Calculate
[1] $110_2 + 11_2 = 1001_2$
[2] $10011_2 + 101110_2 = 1000001_2$
[3] $1110_2 - 101_2 = 1001_2$
[4] $1011001_2 - 100110_2 = 110011_2$
[5] $1110_2 * 10_2 = 11100_2$
[6] $110110_2 * 11110_2 = 11001010100_2$
[7] $1110_2 : 10_2 = 111_2$
[8] $101111011_2 : 1010_2 = 100101$ reminder : 01001_2

Task 2: Hexadecimal number system (Points: 8)
a) Briefly describe how numbers can be converted from the decimal system to the hexadecimal system. Also briefly describe the reverse direction: converting a number from the hexadecimal system to the decimal system.
<u>Answer</u>

Converting decimal to hexadecimal

1. The decimal number(in base 10) is divided by 16.   Get the division as an integer division.
2. Write down the remainder in hexadecimal (0-9,A-F).
3. Iterate the steps until the result is 0.
4. The hex value is the digit sequence of the remainders

Converting hexadecimal to decimal

1. Make the last digit of the hex number as the current digit.
2. Make a variable, let's call it power. Set the value to 0.
3. Multiply the current digit with (16^**power**), store the result.
4. Increment power by 1.
5. Set the current digit to the previous digit of the hex number**.**
6. Iterate the steps until all digits have been multiplied.
7. Sum the result of step 3 to get the answer number.

b) Convert the following numbers from the decimal system to the hexadecimal system. Indicate your calculation method for all subtasks.

[1] $22_{10} = 16_{16}$

Division by 16	Quotient	Remainder	Digit
(22)/16	1	6	0
(1)/16	0	1	1

= $(16)_{16}$

[2] $199_{10} = (12_{10} = C_{16}\ , 7_{10} = 7_{16}\ ) = C7_{16}$

Division by 16	Quotient	Remainder (Digit)	Digit
(199)/16	12	7	0
(12)/16	0	12	1

= $(C7)_{16}$

[3] $4096_{10} = 1000_{16}$

Division by 16	Quotient	Remainder (Digit)	Digit
(4096)/16	256	0	0
(256)/16	16	0	1
(16)/16	1	0	2
(1)/16	0	1	3

= $(1000)_{16}$

[4] $7512{,}56_{10} = 1D58.8F5C28_{16}$

c) Convert the following numbers from the binary system to the hexadecimal system. Indicate your calculation method for all subtasks.

[1] $11011_2 = 1B_{16}$

Convert to decimal = 27

Division by 16	Quotient	Reminder	Bit
27/16	1	11	0
1/16	0	1	1

[2] $1010_2 = A_{16}$

Convert to decimal = 10. And 10 in hex is A

[3] $11011000101_2 = 6C5_{16}$
Convert to decimal = 1733

Division by 16	Quotient	Reminder	Bit
1733/16	108	5	0
108/16	6	12 □ C	1
6/16	0	6	2

[4] $1101101,1001_2 = 6D.9_{16}$

d) Convert the following numbers from the hexadecimal system to the decimal system. Indicate your calculation method for all subtasks.

[1] $2300_{16} = 8960_{10}$
$(2300)_{16} = (2 \times 16^3) + (3 \times 16^2) + (0 \times 16^1) + (0 \times 16^0) = \mathbf{(8960)_{10}}$

[2] $D400_{16} = 54272_{10}$
$(D400)_{16} = (13 \times 16^3) + (4 \times 16^2) + (0 \times 16^1) + (0 \times 16^0) = \mathbf{(54272)_{10}}$

[3] $800_{16} = 2048_{10}$
$(800)_{16} = (8 \times 16^2) + (0 \times 16^1) + (0 \times 16^0) = \mathbf{(2048)_{10}}$

[4] $FF4AE_{16} = 1045678_{10}$
$(FF4AE)_{16} = (15 \times 16^4) + (15 \times 16^3) + (4 \times 16^2) + (10 \times 16^1) + (14 \times 16^0) = \mathbf{(1045678)_{10}}$

Task 3: Polynomials (Points: 10)
a) Briefly describe in your own words what the difference is between a polynomial and a monomial.
Answer
A polynomial is an algebra expression formed by the sum of terms or sum of monomials. Or it can say that monomials are summands of polynomials.
Monomials cannot have an addition or subtraction among the variables.

b) Simplify with polynomial addition:
[1] $(x^2 + 4x + 1) + (x^3 - 5 + 6x) = x^3 + x^2 + 4x + 6x + 1 - 5 = \mathbf{x^3+x^2+10x-4}$
[2] $(yx^4 - xy + 5) + (x^3 - 5yx^4) = yx^4 - 5yx^4 + x^3 - xy + 5 = \mathbf{-4yx^4+x^3-xy+5}$

c) Simplify with polynomial subtraction:
[1] $(x^2 + 4x + 1) - (x^3 - 5 + 6x) = -x^3+x^2+4x-6x+1-5 = \mathbf{-x^3+x^2-2x-4}$
[2] $(yx^4 - xy + 5) - (x^3 - 5yx^4 + 2 + xy) = yx^4 + 5\,yx^4 - x^3 - xy - xy + 5 - 2 = \mathbf{6yx^4 - x^3 - 2xy + 3}$

d) Simplify with polynomial multiplication:
[1] $(4x + y) * (5 + 6x) = 20x + 24x^2 + 5y + 6xy = \mathbf{24x^2 + 6xy + 20x + 5y}$
[2] $(3x^2 - 5x - 2) * (4x - 3) = 12x^3 - 9x^2 - 20x^2 + 15x - 8x + 6 = \mathbf{12x^3 - 29x^2 + 7x + 6}$

e) Simplify with polynomial division:

[1] $(x^3 - 12x - 42) : (x - 3) =$ **$x^2 + 3x - x + \frac{-51}{x-3}$**

[2] $(x^5 - 3x^4 + 2x^3 + 7x^2 - 3x + 5) : (x^2 - x + 1) =$ **$x^3 - 2x^2 - x + 8 +$** $\frac{6x - 3}{x2 - x + 1}$

Task 4: Modulo (Points: 12)

a) Calculate and give your solution path: (6)

mod 10 = {0-9}
mod 3 = {0-2}

[1] 3 mod 10 = **3**
[2] 13 mod 10 = **3**
[3] 23 mod 10 = **3**
[4] 33 mod 10 = **3**
[5] 5 mod 3 = **2**
[6] -5 mod 3 = **1**

b) Calculate and give your solution path: (6)
[1] (9 mod 5 + 31 mod 5) mod 5 **=** (4 + 1) mod 5 = 5 mod 5 **= 0**
[2] (76 mod 10 – 80 mod 10) mod 10 = (6 - 0) mod 10 = 6 mod 10 = **6**
[3] (13 mod 4 * 1 mod 4) mod 4 = (1 * 1) mod 4 = 1 mod 4 = **1**

Task 5: Boolean Algebra and Truth Tables (Points: 15,5)

a) For the following types of logic gates, note the circuit symbols, function, and truth value table: (3,5)
[1] AND
[2] OR
[3] NOT
[4] NAND
[5] NOR
[6] XOR
[7] XNOR

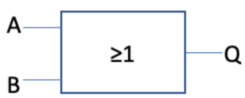
Answer

AND □ Gives a high output (1) only if all its inputs are high

Truth Table

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

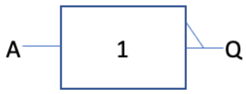
OR □ Gives a high output (1) if one or more of its inputs are high.



Truth Table

A	B	OR
0	0	0
0	1	1
1	0	1
1	1	1

NOT □ Or also known as *inverter* produces an inverted version of the input at its output.



Truth Table NOT

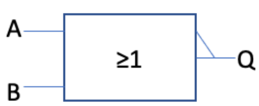
A	Q
0	1
1	0

NAND □ A NOT-AND gate, produces an output which is false only if all its inputs are true.

Truth Table

A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0

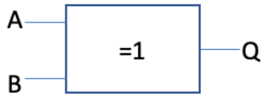
NOR □ A NOT-OR gate, Give a high output (1) if both inputs are low (0).



Truth Table

A	B	NOR
0	0	1
0	1	0
1	0	0
1	1	0

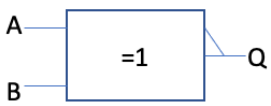
XOR □ Gives a true (1 or HIGH) output when one of the input (not both) is 1.



Truth Table

A	B	XOR
0	0	0
0	1	1
1	0	1
1	1	0

XNOR □ Giving High output (1) if and only if both of inputs are same.



Truth Table

A	B	XNOR
0	0	1
0	1	0
1	0	0
1	1	1

b) Boolean algebra satisfies many of the same laws as ordinary algebra when one matches up ∨ with addition and ∧ with multiplication. What are the following laws concerning boolean algebra: (4)

Premise:
p,q,r are variables
T = Tautology
F = Contradiction

[1] Associativity of ∨
Answer
p ∨ (q ∨ r) <==> (p ∨ q) ∨ r

[2] Associativity of ∧
Answer
p ∧ (q ∧ r) <==> (p ∧ q) ∧ r

[3] Commutativity of ∨
Answer
p ∨ **q** <==> **q ∨ p**

[4] Commutativity of ∧
Answer
p ∧ **q** <==> **q ∧ p**

[5] Identity of ∨
Answer
p ∨ F <==> P

[6] Identity of ∧
Answer
q ∧ F <==> P

[7] Distributivity of ∧ over ∨
Answer
p ∧ (q ∨ r) <==> (p ∧ q) ∨ (p ∧ r)

[8] Annihilator for ∧
Answer
p ∧ F <==> F

c) Prove De Morgan's laws with the help of truth value tables. (2).

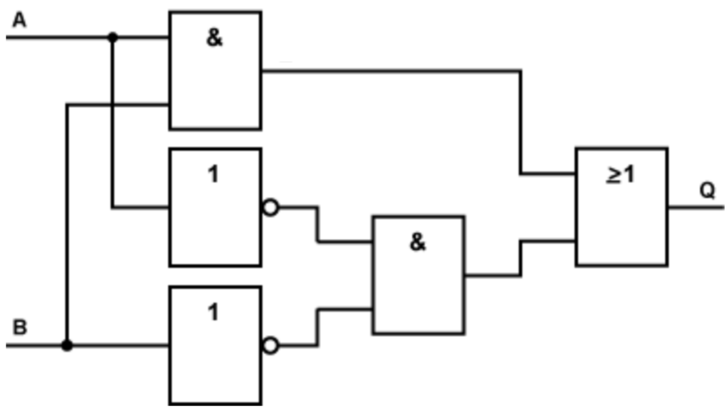
[1] $\underline{a \wedge b} = \underline{a} \vee \underline{b}$

a	b	a ∧ b	$\underline{a \wedge b}$	$\underline{a}$	$\underline{b}$	$\underline{a} \vee \underline{b}$
0	0	0	1	1	1	1
0	1	1	1	1	0	1
1	0	1	1	0	1	1
1	1	1	0	0	0	0

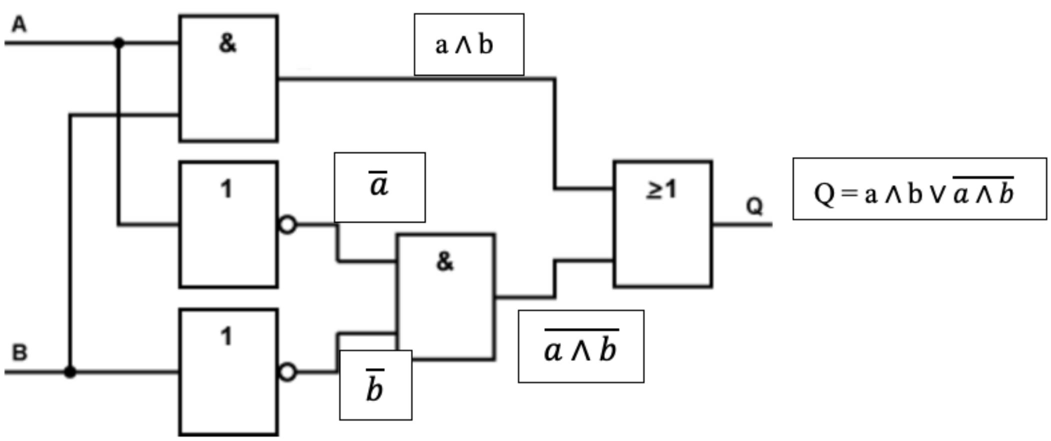
[2] $\underline{a \lor b} = \underline{a} \land \underline{b}$

a	b	a ∨ b	$\underline{a \lor b}$	$\underline{a}$	$\underline{b}$	$\underline{a} \land \underline{b}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

d) Look at the circuit diagram. Develop a corresponding propositional formula for the given circuit diagram. Also draw up a corresponding truth value table. What do you notice? (6)



Answer



Q = a ∧ b ∨ $\underline{a \land b}$

a	b	a ∧ b	$\underline{a \land b}$	Q = a ∧ b ∨ $\underline{a \land b}$
0	0	0	1	1
0	1	0	1	1
1	0	0	1	1
1	1	1	0	1